SACAI

EXAMINATION GUIDELINES

MATHEMATICS

(REVISED)

GRADE 12

2015/2016

SOUTH-AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE
SUID-AFRIKAANSE KOMPREHENSIEWE ASSESSERINGSINSTITUUT

EXAMINATION GUIDELINES FOR NSC EXAMINATION: GRADE 12

CONTENTS

1. INTRODUCTION

SACAI aims to provide an assessment and examination service of quality up to Level 4 of the National Qualifications Framework (NQF). Therefore SACAI aims at the maintaining of high academic standards within the relevant policy framework for Mathematics.

The purpose of this document is to provide some clarity on the depth and outlook of the Mathematics Content to be assessed in the Grade 12 National Senior Certificate Examination (NSC). SACAI aims to assist teachers so that they can prepare the learners sufficiently for the NSC Examinations.

This guideline document should be read in conjunction with the following:

A. The National Curriculum Statement (NCS):

- *Curriculum and Assessment Policy Statement (CAPS):* **Mathematics**
- The National Protocol for Assessment Grades R – 12.
- The National Policy pertaining to the programme and promotion requirements of the National Curriculum Statement, Grades R-12

B. The *Revised SACAI/CAPS Subject Guidelines (2015)*

2. ASSESSMENT IN GRADE 12

2.1 **Format of the question papers**

The Grade 12 NSC-Mathematics Examination consists of 2 compulsory question papers.

Questions in both Papers 1 and 2 will assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of contexts.

This examination is externally set, marked and moderated.

The final examination makes up 75% of the total mark for Mathematics.

All work done in Grade 11 Mathematics is examinable, although the work will not be covered again in Grade 12.

For the benefits of the learners, it is important and strongly encouraged that the content and reasoning required for NSC-Examination be finished as soon as possible.

The Preparatory Examination must be closely related to the final examination in terms of weighting (marks allocated) of contents, time and layout of papers.

In the June exams, Preparatory exams and Final exams, learners will be examined on the entire Grade 11 curriculum in Mathematics. Grade 11 work will form a big part of the June Examinations and learners need to spend time on revising Grade 11 work.

****A formulae sheet must be provided for each exam****

2.2 Weighting of the Cognitive Levels

The different cognitive levels of the Mathematic questions in the exams, will be according to the Taxonomy table shown below.

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge 20% (30 marks per question paper)	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	1 Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ 2 The angle AOB subtended by arc AB at the centre O of a circle ….
Routine procedures 35 % (52 – 53 marks per question paper)	• Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formulae on the information sheet(no changing of the subject) • Perform well-known procedures • Simple applications and calculations which might involve steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	Solve for x: $x^2 - 5x = 14$ Determine the general solution of the equation $2\sin(x - 30^\circ) + 1 = 0$ Prove that the angle AOB subtended by arc AB at The centre O of a circle is double the size of the angle ACB which the same arc subtends at the circle.
Complex procedures 30% (45 marks per question paper)	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding • Learners need to be able to solve problems by integration of different sections.	What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is100km/ h and the average speed for the return journey is 80km/ h? Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x.

Problem Solving 15% (22 – 23 marks per question paper)	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts. • Interpretation of the solution find for the problem by solving of problems of which the contexts is unknown.	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not?

Make sure that the learners are familiar with the skills that they will be examined on.

Learners need to be familiar with these skills on <u>all Assessments</u> for the whole year

2.3 Time and Mark allocation

The Question papers will be structured according to the weighting shown in the table below.

Description	**Marks**
Question paper 1: **Grade 12: Book work, e.g. proofs of formulae (Maximum of 6 marks)**	
Algebra equations and inequalities	25 ± 3
Patterns and sequences	25 ± 3
Finance, growth and decay	15 ± 3
Functions and graphs	35 ± 3
Differentiation	35 ± 3
Probability	15 ± 3
Total Marks	**150**
Duration	**3 hours**
Description	**Marks**
Question paper 2: **Grade 12: Book work, e. g. Euclidian Geometry theorems and Trigonometry formulae (Maximum of 12 marks)**	
Statistics and Regression	20 ± 3
Analytical Geometry	40 ± 3
Trigonometry	40 ± 3
Euclidean Geometry and Measurement	50 ± 3
Total Marks	**150**
Duration	**3 hours**

Note:

It is compulsory to include book work: learners must know how to derive formulae as stipulated in the Curriculum for examination purposes, for both question papers.

Question Paper 1: a maximum of 6 marks are allowed for book work.

Question paper 2: a maximum of 12 marks are allowed for book work.

The mark allocation may be explained as follows: The ± 3 means that the total marks for the specific section may be 3 marks less or 3 marks more.

E.g. Take *Probability* 15 ± 3.The desired mark is 15 but you are allowed to have a mark of 12 (3 marks less) or a mark out of 18 (3 marks more).

NOTE:

- ✓ Questions will not be necessarily be given in sections as the table above indicates.
- ✓ Various topics can be integrated into one question.
- ✓ Mathematic modeling as a process will be included in both papers. That means that 'word sums' can be expected as questions in any of the topics.

2.4 Term tests and Tasks

	GRADE 12	
	Tasks and weighting	**%**
TERM 1	**Project/investigation** **Formal Test** **Assignment**	**20** **10** **10**
TERM 2	**Formal Test** **Mid-year examination**	**10** **15**
TERM 3	**Formal TEST** **TRIAL EXAMS**	**10** **25**
TERM 4	**External exams**	
SBA Mark for the year		**100**
SBA mark as percentage of promotion mark		**25%**
End -of -year exams		**75%**
Promotion mark		**100%**

- *Formal Tests should be 1 hour long and count 50 marks.*

3. Elaboration of the Content for Grade 12 (CAPS)

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as learners should understand Mathematics as a holistic discipline.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process learners must be able to understand how f (x) has been transformed to generate f (−x), − f (x) , f (x + a) , f (x) + a , af (x) and x = f (y) where a ∈R.
2. Trigonometric functions will ONLY be examined in Paper 2, but trigonometric expressions and properties may be used in paper 1.

NUMBER PATTERNS, SEQUENCES AND SERIES

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FINANCE, GROWTH AND DECAY

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating for *i* in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will not be examined in the examination.

ALGEBRA

1. Solving quadratic equations by completing the square will not be examined. Learners may still apply it to solve for x
2. Solving quadratic equations using the substitution method (k-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

DIFFERENTIAL CALCULUS

1. The following notations for differentiation can be used: f ′(x), Dx , dydx or y′.
2. In respect of cubic functions, candidates are expected to be able to:

    - Determine the equation of a cubic function from a given graph.
    - Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
    - Apply knowledge of transformations on a given function to obtain its image.

3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will not be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

PROBABILITY

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced, is examinable.
3. Counting principles, where the arrangement is circular and/or using combinations, is not in the spirit of the curriculum.
4. In word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
    - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
    - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
    - The opposite angles of a cyclic quadrilateral are supplementary;
    - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
    - that a line drawn parallel to one side of a triangle divides the other two sides proportionally;
    - equiangular triangles are similar.
    - Similar triangles are equiangular.
    - Prove the Pythagorean theorem using similarity (or A line from the right angle perpendicular to hypotenuse, divides the triangle into three similar triangles)
4. Corollaries derived from the theorems and axioms are necessary in solving riders:

    - Angle in a semi-circle
    - Equal chords subtend equal angles at the circumference
    - Equal chords subtend equal angles at the centre

- In equal circles, equal chords subtend equal angles at the circumference
- In equal circles, equal chords subtend equal angles at the centre.
- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
- If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- Tangents drawn from a common point outside the circle are equal in length.
- The line that cuts two sides of a triangle proportionally is parallel to the third side.

5. The theory of quadrilaterals will be integrated into questions in the examination.

6. Concurrency theory is excluded.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinear points must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.

4. Euclidian Geometry: Accepted reasons

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt OR ∠s in a revolution
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	Co int ∠s supp

TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ OR sum of ∠s in Δ OR Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras OR Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras OR Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS OR S∠S

If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS OR ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS OR 90°HS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ OR prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs OR equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height OR equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi-circle **OR** diameter subtends right angle **OR** $\angle\frac{1}{2}$

If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi-circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are on the circumference of the circle.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad sup **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common ptOR Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord

QUADRILATERALS	
The interior angles of a quadrilateral add up to 360◦.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ||
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ||m

If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral is equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

5. Marking Guidelines

5.1 If a learner answers a question more than once and does not cancel any of them out, the first attempt must and will be marked irrespective of which one have the correct answer

5.2 Consistent accuracy marking must be followed when e.g. a certain variable is wrongly calculated in one sub-question and needs to be used in another sub-question. Full marks may be given for the subsequent sub-question provided that the methods and calculations used are correct.

6. **Formula sheet**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)A = P(1 - ni)A = P(1 - i)^n A = P(1 + i)^n$$

$$F = \frac{x[(1 + i)^n - 1]}{i} P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$T_n = a + (n - 1)dS_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1} S_n = \frac{a(r^n - 1)}{r - 1} S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} M = \left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + cy - y1 = m(x - x1)m = tan\theta(x - a)^2 + (y - b)^2 = r$$

$$\text{In}\Delta ABC: \frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} a^2 = b^2 + c^2 - 2abcosA \; Area\Delta ABC = \frac{1}{2} absinC$$

$$sin(\alpha + \beta) = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta sin(\alpha - \beta) = sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta$$

$$cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta cos(\alpha - \beta) = cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta$$

$$P(AofB) = P(A) + P(B) - P(AenB) \acute{x} = \frac{\sum fx}{n} \sigma^2 = \frac{\sum_{i=1}^{n} (x_1 - \acute{x})^2}{n}$$

$$\hat{y} = a + bx \text{where the gradient } b = \frac{\sum(x - \acute{x})(y - \acute{y})}{\sum(x - \acute{x})^2}$$

7. **Conclusion**

This Examination Guidelines document is meant to be used as a support tool for educators/tutors in teaching Mathematics Gr 12. It is aimed at supplying educators/tutors with more information on what to expect from learners when preparing for tests and examinations.

It is therefore not a substitute for the CAPS document which Educators should teach to. It is important that these Guidelines must be used in correspondence with the *Revised SACAI CAPS Guidelines: Mathematics.*